Linear Analysis of Tumor Morphology during Growth using Bending Energy

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The need for mathematical models in the field of oncology has been widely addressed by scientists. The ability of tumors to metastasize is mostly preceded by morphological instabilities. Therefore, parameters that describe tumor morphology during growth may also characterize tumor's invasive ability. This work studies the evolution of critical parameters that relate to the morphological stability of tumor spheroids. The interaction between a non-circular tumor and its host is analyzed using a two-phase Stokes model. The Stokes flow is used as a constitutive law to model tissue stresses. The model assumes that the tumor and host are treated as incompressible viscous fluids, and that the tumor cell population is homogeneous. Tumor growth is assumed to be regulated by externally supplied cell substrates, such as oxygen, through cell proliferation (mitosis) and apoptosis. In this two-dimensional model, the pressure that occurs due to cell proliferation, acts as an expansion force. Morphological changes are evaluated using the Helfrich bending energy, which provides input regarding the stiffness of the interface between tumor and the host. Using the bending energy approach, a modified Young-Laplace equation for the stress jump across the interface is developed through an energy variation approach. A non-dimensionalization process leads to parameters that assess the effects of viscosity, bending rigidity, and apoptosis on tumor morphology. The suitability of the system parameters, as predicted by the model is analyzed using a linear stability and a sensitivity analysis. These effects are analyzed during the avascular and vascular state. The two-phase Stokes model suggests that the bending rigidity, viscosity and apoptosis are parameters that influence tumor shape morphology. Linear analysis suggests that during the avascular stage, increased bending rigidity contributes to increased morphological stability, increased apoptosis promotes shape instabilities, an increase in vascularization leads to a stable morphology for small radial values and a more self-similar growth for large tumors almost independently of the viscosity values, and that increased tumor viscosity versus host viscosity leads to unstable behavior. Findings regarding tumor viscosity are consistent with previously conducted studies using a single-phase model. Sensitivity analysis suggests that bending rigidity factor is an influential parameter on tumor morphology during growth.